Correct conclusion

The student’s final statement — that the quadrilateral formed by joining the midpoints of a square’s sides is a square — is correct. However the explanation given is incomplete and unclear. Below is a clear, step-by-step justification and feedback.

Step-by-step proof (coordinate method)

1. Place the square on coordinates: let A=(0,0), B=(1,0), C=(1,1), D=(0,1).
2. Find midpoints of consecutive sides: M_AB=(0.5,0), M_BC=(1,0.5), M_CD=(0.5,1), M_DA=(0,0.5).
3. Compute vectors for consecutive edges of the midpoint quadrilateral:
   v1 = M_AB→M_BC = (0.5,0.5),
   v2 = M_BC→M_CD = (-0.5,0.5).
4. Lengths: |v1| = sqrt(0.5^2+0.5^2) = sqrt(0.5). |v2| = same. So all four sides are equal (it is a rhombus).
5. Dot product of adjacent vectors: v1·v2 = (0.5)(-0.5)+(0.5)(0.5)=0. So adjacent sides are perpendicular, therefore the shape is a square.

Short conceptual justification

Varignon’s theorem: joining midpoints of any quadrilateral gives a parallelogram. For a square those midpoints give equal adjacent side lengths and right angles, so the parallelogram is a square (rotated 45° relative to the original).

Evaluation of the student answer

Student answer: “a quadrilateral has 4 sides, the only quadrilateral to be be made from wxyz's midpoints that is the square”

- Correct final claim: yes.
- Missing justification: yes — no use of Varignon’s theorem, coordinates, vectors, or diagram.
- Communication problems: grammar and clarity issues; doesn’t explain why a square must result.

Teacher comments (strict, clear — Amy Chua cadence, ~200 words)

You are right to identify the final shape as a square. That is the only important part you got completely correct — and you must be louder about what you know and quieter about what you guess. But right now you handed me an answer that is a claim without a single reason. Mathematics is proof; it is not opinion. When I ask for justification, I expect a diagram or an algebraic argument, not a restatement of what a quadrilateral is. Redo this: draw the original square, mark midpoints clearly, label coordinates or write the vector relations. Show that opposite sides are parallel (Varignon) and then show equal adjacent side lengths and a right angle. Neat handwriting, labelled points, and stepwise logic. If you struggle, write one line at a time and check each line for truth. Be precise with language: say "midpoint of AB" rather than "wxyz's midpoints." I accept a short coordinate proof or a clean diagram plus reasoning. Until you show that reasoning, a correct final word counts for very little. Now go and rewrite this properly — I expect improved clarity and a complete proof next time.

ACARA v9 mapping

Relevant content: Geometry — properties of shapes and quadrilaterals; use of coordinate geometry and vector reasoning to prove geometric results. (Relevant to Years 7–8 ACARA v9 learning sequence on shapes and reasoning about geometric relationships.)

Rubric (10 marks)

1. **Understanding (0–3)**: 3 = states correct result; 2 = correct claim but incomplete; 1 = wrong claim; 0 = no attempt.
2. **Reasoning/Proof (0–3)**: 3 = clear proof (coordinates, vectors, or Varignon); 2 = partial reasoning; 1 = assertion with wrong/irrelevant step; 0 = none.
3. **Mathematical Communication (0–2)**: 2 = correct labels, notation and sentences; 1 = some unclear notation/grammar; 0 = unreadable.
4. **Use of diagram/working (0–2)**: 2 = neat diagram or calculations supporting proof; 1 = sketch only; 0 = none.

Example scoring for this student

Understanding: 2 (correct claim). Reasoning: 0 (no proof). Communication: 1 (unclear phrasing). Diagram/working: 0. Total = 3/10. Feedback: provide a labeled diagram or coordinate proof to raise score.

How to improve (steps)

1. Draw the square and label A, B, C, D and midpoints M1–M4.
2. Either use Varignon’s theorem or place coordinates to show the result algebraically.
3. Write short concluding sentence: "Therefore the midpoint quadrilateral is a square because..." and cite perpendicular adjacent sides and equal lengths.