The Pythagorean Detective: Uncovering a Secret Right-Angle Formula

Materials Needed:

- Graph paper (1 sheet)
- A pencil and an eraser
- A ruler
- Scissors
- A calculator
- About 30 small, square objects (Cheese crackers are perfect and make for a good snack! You can also use 1cm x 1cm paper squares.)

Learning Objectives

By the end of this lesson, you will be able to:

- **Explain** the Pythagorean theorem in your own words.
- **Prove** the theorem visually using a hands-on model.
- **Apply** the formula $a^2 + b^2 = c^2$ to solve for unknown distances in real-world scenarios.
- **Create** a unique project (a design, story, or explanation) that demonstrates your understanding of the theorem.

Curriculum Standard

This lesson aligns with Common Core State Standards for Mathematics (CCSS.MATH.CONTENT.8.G.B.7): Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Lesson Activities

Part 1: The Case of the Missing Shortcut (5 Minutes)

Let's start with a mystery! Imagine you're standing at one corner of a rectangular park. The park is 80 meters long and 60 meters wide. You need to get to the diagonally opposite corner.

You can walk along the path on the edges (the long side and then the short side), or you can take a shortcut and walk straight across the grass. **The question is: exactly how much shorter is the shortcut?**

Don't solve it yet! Just think about it. We have a triangle, but one side's length is a mystery. We need a special tool to solve this, and that's what we're going to uncover today.

Part 2: Cracking the Code with Crackers! (15 Minutes)

This is where you become a real math detective and discover the secret formula for yourself.

1. On your graph paper, use your ruler to draw a right-angled triangle. Make the two sides that form the right angle specific lengths: one side should be **3 units** (3 squares on the paper) long, and the other should be **4 units** long.
2. Connect the ends to form the third, longest side. This side is called the **hypotenuse**.
3. Now, let's build squares using our crackers (or paper squares). On the side that is 3 units long, build a square that extends outwards from the triangle. How many crackers did it take? (It should be 3x3 = 9).
4. Do the same for the side that is 4 units long. How many crackers did it take? (It should be 4x4 = 16).
5. Finally, carefully build a square on the hypotenuse. You'll have to tilt the crackers to make them fit. Count how many crackers it took to build this largest square.

The Big Discovery: You should find that it takes exactly **25 crackers** to build the square on the hypotenuse. Now look at the numbers. What do you notice about 9, 16, and 25?

That's right! 9 + 16 = 25.

You just visually proved the Pythagorean theorem! You've shown that the square of the first side plus the square of the second side equals the square of the hypotenuse.

Part 3: The Big Reveal: The Formula (5 Minutes)

Mathematicians write down this "cracker discovery" with a famous formula. For any right-angled triangle:

$$a^2 + b^2 = c^2$$

- **a** and **b** are the two shorter sides that form the right angle (they are called the "legs").
- **c** is always the longest side, the one opposite the right angle (the "hypotenuse").

Part 4: Real-World Detective Work (20 Minutes)

Now that you have the secret formula, let's use it to solve some cases!

1. **Case #1: The Park Shortcut.** Let's go back to our park. The sides are a=60 meters and b=80 meters. Let's find 'c', the shortcut distance.
    - $a^2 + b^2 = c^2$
    - $60^2 + 80^2 = c^2$
    - $3600 + 6400 = c^2$
    - $10000 = c^2$
    - To find 'c', we need the square root of 10000. Use your calculator! $\sqrt{10000} = 100$.
    - **Solution:** The shortcut is 100 meters. The path was 60+80 = 140 meters. You saved 40 meters of walking!
2. **Case #2: The Ladder Dilemma.** A firefighter has a 15-foot ladder. To be safe, the base of the ladder must be placed 4 feet away from the wall. How high up the wall can the ladder reach? (*Hint: The ladder is the hypotenuse, 'c'. You're solving for 'b'.*)
3. **Case #3: The Video Game Quest.** You are designing a video game map. Your character needs

to get a treasure that is 16 blocks east and 12 blocks south of their current position. What is the shortest possible distance the character's "teleport" spell would need to cover?

Assessment: The Creative Challenge (15-20 Minutes)

Instead of a boring quiz, show me you understand the theorem by choosing **one** of the following creative projects.

- **Option A: The Architect**

  Design a blueprint on graph paper for something cool that uses right triangles. It could be a skate ramp, a treehouse with a zip line, or a support structure for a bridge. Clearly label the lengths of the sides of at least two different right triangles, proving your calculations are correct with the Pythagorean theorem.

- **Option B: The Game Designer**

  Create a simple treasure map on paper. The map should have a starting point and an X for the treasure. The instructions to find the treasure can't be direct—they must be a puzzle that can only be solved by calculating the hypotenuse of a right triangle. For example: "Start at the old oak, walk 9 paces north, then 12 paces west. The treasure is buried directly under the spot that is [your calculated hypotenuse] paces from the old oak."

- **Option C: The Explainer**

  Create a one-page "cheat sheet" or comic strip that would teach a friend about the Pythagorean theorem. It must include: what 'a', 'b', and 'c' are; the formula; and at least one drawn example of how it can be used to solve a real problem.

Extension (Optional Challenge)

A "Pythagorean Triple" is a set of three whole numbers that work perfectly in the theorem (like our 3, 4, 5 triangle). Can you find another set of three whole numbers that works? Try starting with a leg ('a') of 5 and see if you can find a 'b' and 'c' that are whole numbers!