

Operation Detectives: Finding the Hidden Variable (X)

Materials Needed

- Paper or Notebook
- Pencil or Pen
- Index Cards or small slips of paper (approximately 10)
- Calculator (optional, for checking answers)
- Optional: Markers or highlighters to circle variables and operations

Learning Objectives (SWBAT)

By the end of this lesson, learners will be able to:

1. Accurately apply the standard Order of Operations (PEMDAS/GEMDAS) to simplify expressions.
2. Understand and apply the inverse Order of Operations (SADMEP) to isolate and solve for an unknown variable (x).
3. Successfully solve multi-step equations involving up to four different operations.
4. Verify the solution by substituting the found variable back into the original equation.

Introduction: The Mathematical Mystery Box

Hook (5 minutes)

Imagine you have a magic box, and inside that box is a secret number. You tell your friend to perform three specific operations on the box's contents: multiply it by 2, add 5, and then divide the whole thing by 3. Your friend says the final answer is 7. How do you figure out the number that was in the box before they started?

That secret number is our variable, 'x.' Today, we are learning to become mathematical detectives who can unwrap complex equations to find the hidden value of the unknown variable. We use the Order of Operations to build equations, but we use the *reverse* Order of Operations to solve them!

Success Criteria

You will know you are successful when you can correctly solve three out of four complex equations for 'x' and show your steps clearly.

Body: Building and Unwrapping Equations

Phase 1: Reviewing the Tools (I Do) (10 minutes)

Topic: Forward Order of Operations (PEMDAS/GEMDAS)

Before we can undo an equation, we need to remember the rule for building it. This rule ensures everyone gets the same answer when simplifying an expression:

- **P/G:** Parentheses / Grouping Symbols
- **E:** Exponents
- **MD:** Multiplication and Division (from left to right)
- **AS:** Addition and Subtraction (from left to right)

I Do: Modeling Simplification

Example Expression: $4 * (2 + 3)^2 - 10$

1. (P) Solve inside parentheses: $4 * (5)^2 - 10$
2. (E) Solve exponent: $4 * 25 - 10$
3. (MD) Multiply: $100 - 10$
4. (AS) Subtract: 90

(Educator Note: This is crucial. Learners must be confident in PEMDAS before tackling the reverse process.)

Phase 2: Reverse Order of Operations (SADMEP/Unwrapping the Gift) (15 minutes)

Topic: Isolating the Variable

When we solve for 'x,' we are doing the opposite of PEMDAS. We are "unwrapping" the equation. We must remove the operations farthest away from the variable first.

We use **SADMEP** (the reverse acronym) to decide which operation to undo first:

- **AS:** Addition and Subtraction (First to undo)
- **MD:** Multiplication and Division (Second to undo)
- **E:** Exponents (Third to undo)
- **P/G:** Parentheses / Grouping (Last to undo)

I Do: Modeling Solving for X

Let's solve the equation: $3x - 12 = 30$

1. **Identify the operations:** x is multiplied by 3, then 12 is subtracted.
2. **Step 1 (SADMEP: AS):** Undo Subtraction. The opposite of subtracting 12 is adding 12.
 - $3x - 12 + 12 = 30 + 12$
 - $3x = 42$
3. **Step 2 (SADMEP: MD):** Undo Multiplication. The opposite of multiplying by 3 is dividing by 3.
 - $3x / 3 = 42 / 3$
 - $x = 14$
4. **Verification (Check):** Substitute 14 back into the original equation using PEMDAS.
 - $3(14) - 12 = 42 - 12 = 30$. (It works!)

Phase 3: Guided Practice (We Do) (15 minutes)

Complex Equation: $(x / 4) + 6 = 15$

Instructions: Solve this equation step-by-step. Write down the operation you are undoing on each line.

1. What is the operation farthest away from x? (Addition of 6)

2. How do we undo adding 6? (Subtract 6 from both sides)
 - $(x / 4) + 6 - 6 = 15 - 6$
 - $x / 4 = 9$
3. What is the remaining operation? (Division by 4)
4. How do we undo dividing by 4? (Multiply both sides by 4)
 - $(x / 4) * 4 = 9 * 4$
 - $x = 36$
5. **Verification:** $(36 / 4) + 6 = 9 + 6 = 15$.

Phase 4: Independent Practice & Application (You Do) (20 minutes)

Activity: Variable Swap Challenge

This challenge provides hands-on practice in both building complex equations and solving them.

Setup:

1. On a piece of scratch paper, choose an integer value for X (e.g., $X = 8$). Keep this secret!
2. Using this secret X value, create a complex multi-step equation using at least three different operations (e.g., $5(x + 2) - 15 = ?$).
3. Solve your own equation using your secret X to find the final result. (e.g., $5(8 + 2) - 15 = 5(10) - 15 = 50 - 15 = 35$).
4. Write ONLY the equation with the final answer on an index card, replacing the secret number with 'x'. (Card reads: $5(x + 2) - 15 = 35$).

The Solve:

1. Swap your index card with a partner (or the educator/parent).
2. Using SADMEP, solve the mystery equation you received.
3. When you have solved it, check your answer by substituting your result back into the original equation.
4. Compare your result with the creator's secret X value. (Did you find the same number?)

Adaptability Note:

- **Classroom:** Students partner up and swap cards.
- **Homeschool/Training:** The student creates the problem for the educator/parent, and the educator creates a problem for the student, or the student simply completes three challenges independently.

Conclusion: Reflecting on the Process

Recap and Discussion (5 minutes)

Let's summarize our detective work:

- When we simplify an expression, we use _____. (PEMDAS)
- When we solve for an unknown variable, we must reverse the order and use _____. (SADMEP)
- Why is it important to perform the same operation on both sides of the equal sign? (To keep the equation balanced.)

Formative Assessment: Quick Check (5 minutes)

On a half-sheet of paper, solve the following problem and show your first two steps clearly.

Challenge Question: $(x / 5) - 3 = 7$

(Check for correct identification of the first operation to undo (Subtracting 3, so add 3) and the second (Dividing by 5, so multiply by 5). Correct answer: $x=50$)

Differentiation and Flexibility

Scaffolding (Support for Struggling Learners)

- **Visual Aid:** Provide a laminated SADMEP checklist that learners can reference or write on with a dry-erase marker.
- **Step Reduction:** Focus only on two-step equations (e.g., $Ax + B = C$) until mastery is achieved before introducing parentheses or division.
- **Inverse Practice:** Have the learner verbally state the inverse operation before writing it down (e.g., "I see division by 4, so I must multiply both sides by 4").

Extension (Challenge for Advanced Learners)

- **Variables on Both Sides:** Introduce equations where the variable appears in two places (e.g., $4x + 10 = x - 5$). Learners must first use inverse operations to gather all 'x' terms onto one side.
- **Word Problem Translation:** Challenge the learner to write and solve a real-world scenario that translates into a multi-step equation, such as: "Three friends split the cost of a rental car and an \$18 gas fee. If each person paid \$45, what was the original cost (x) of the car rental?" (Equation: $(x + 18) / 3 = 45$)

Summative Assessment (Final Outcome Check)

Task: The Budget Planner

You have saved \$150 for a new video game console (X). You spend \$30 on accessories first. Then, you realize you need three times the amount of money you have left to buy the console. Write and solve an equation to determine the exact cost (X) of the console.

*Expected Equation: $3 * (X - 30) = 150$. Solution: $X = \$80$. (Requires SADMEP application: Divide by 3 first, then Add 30.)*