

# Real-World Trig: Measuring the Impossible

## Materials Needed

- Scientific calculator (must have SIN, COS, TAN functions)
- Protractor (or digital protractor/inclinometer app like Clinometer)
- Measuring tape or ruler (for initial base measurements)
- Worksheet/Notebook and pencil
- Printed or digital copy of the SOH CAH TOA guide/flowchart

## Learning Objectives (SWBAT)

By the end of this lesson, learners will be able to:

1. Accurately recall and identify the three basic trigonometric ratios (Sine, Cosine, Tangent).
2. Apply the appropriate trigonometric ratio (SOH CAH TOA) to find unknown side lengths or angles in right triangles.
3. Construct and solve at least two real-world problems involving right triangles, demonstrating the practical use of trigonometry.

## Success Criteria

You will know you are successful if you can:

- Correctly set up the equation for a given problem (e.g., writing  $\tan(45^\circ) = \text{height} / \text{distance}$ ).
- Calculate the unknown value (side or angle) with less than 5% error.
- Clearly explain why a specific ratio (Sin, Cos, or Tan) was chosen for a problem.

## Lesson Duration

90 Minutes (Flexible: Can be split into two 45-minute sessions)

## Phase 1: Introduction - The Invisible Tape Measure

### Hook (5 minutes)

**Ask:** Imagine you are standing 50 feet away from the base of the tallest roller coaster in the world. You look up at the highest point, and the angle from your eye to the top is 65 degrees. How tall is that coaster? We can't just climb up with a tape measure—that's dangerous and impractical. Today, we're going to learn how to use math to measure things that are too tall, too far, or impossible to reach directly.

### Review of Prerequisite Knowledge (5 minutes)

We are focused entirely on right triangles today. Let's quickly review the vocabulary essential for trigonometry:

- **Right Angle:** The 90-degree corner.
- **Hypotenuse:** The side opposite the right angle (always the longest side).

- **Opposite Side:** The side across from the angle we are interested in.
- **Adjacent Side:** The side next to the angle we are interested in (not the hypotenuse).

## Stating Objectives (5 minutes)

Today, we're putting SOH CAH TOA to work. We are moving beyond just solving triangles on paper and applying these powerful ratios to solve real-life surveying problems.

## Phase 2: Content Delivery and Guided Practice

### I Do: Modeling the Setup (15 Minutes)

#### Topic: Using Trigonometry to Find a Side Length (Angle of Elevation)

**Scenario:** A construction crew needs to know the height of a small building so they can order materials for the roof. They measure the angle of elevation from 100 feet away from the base to be 30 degrees.

1. **Draw and Label:** Sketch the right triangle. The building is the vertical side (Opposite), the distance on the ground is the Horizontal side (Adjacent), and the angle of  $30^\circ$  is our reference angle. We need to find the height (h).
2. **Choose the Ratio (SOH CAH TOA):** We know the Adjacent side (100 ft) and we want to find the Opposite side (h). The ratio that connects Opposite and Adjacent is Tangent (TOA).
3. **Set up the Equation:**  $\tan(\text{angle}) = \text{Opposite} / \text{Adjacent}$   $\tan(30^\circ) = h / 100$
4. **Solve:**  $h = 100 * \tan(30^\circ)$  (Use calculator:  $\tan(30^\circ) \approx 0.577$ )  $h \approx 100 * 0.577$   $h \approx 57.7$  feet (The building is about 58 feet tall).

**Transition:** Now that we've seen how to find a side, let's look at finding an unknown angle.

#### Topic: Using Inverse Trigonometry to Find an Angle

**Scenario:** A wheelchair ramp must be safe. Regulations state the angle cannot be steeper than 5 degrees. The ramp is 20 feet long (Hypotenuse) and rises 3 feet high (Opposite). Is the ramp compliant?

1. **Draw and Label:** The length of the ramp is the Hypotenuse (20 ft). The rise is the Opposite side (3 ft). We want the angle ( $\theta$ ).
2. **Choose the Ratio:** We know Opposite and Hypotenuse. This requires Sine (SOH).
3. **Set up the Equation:**  $\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$   $\sin(\theta) = 3 / 20$   $\sin(\theta) = 0.15$
4. **Solve (Using Inverse):** To find the angle, we must use the inverse sine function ( $\sin^{-1}$ ).  $\theta = \sin^{-1}(0.15)$  (Use calculator:  $\theta \approx 8.63^\circ$ )

**Conclusion:** Since  $8.63^\circ$  is greater than the  $5^\circ$  limit, the ramp is too steep.

### We Do: Collaborative Application (20 Minutes)

#### Activity: The Cable Car Challenge (Think-Pair-Share/Guided Calculation)

**Scenario:** A new cable car route needs to be established. The landing station is 5,000 feet away horizontally from the base of the mountain. The mountain summit is 3,500 feet higher than the landing station. What angle (of incline) must the cable car operate at?

#### Steps for Learners:

1. (Think) Sketch the triangle and identify the known values (Opposite and Adjacent).

- (Pair) Discuss with a partner (or instructor/mentor): Which ratio links Opposite and Adjacent? (Tangent).
- (Share) Set up the equation and solve for the unknown angle using the inverse function.

### Expected Solution:

- $\tan(\theta) = 3500 / 5000 = 0.7$
- $\theta = \tan^{-1}(0.7) \approx 35$  degrees

**Formative Assessment Check:** Circulate and observe setups. Ask learners: "If the Hypotenuse (cable car distance) was given instead of the horizontal distance, what ratio would you use?" (Answer: Sine or Cosine, depending on the second piece of data).

## You Do: Independent Practice & Project (25 Minutes)

### Activity: The Real-Life Surveyor Project

Learners will now step outside (or use known dimensions of a large item indoors, like a tall bookshelf or wall) and apply trigonometry to measure something they cannot easily measure directly.

### Instructions:

- Identify Target:** Choose a tall, fixed object (e.g., a tree, a utility pole, the corner of a house/room).
- Collect Data (Required):**
  - Measure the horizontal distance (the Adjacent side) from your observation point to the base of the object.
  - Use a protractor/inclinometer app to measure the Angle of Elevation (the angle from the ground to the top of the object).
- Calculate Unknown Side:** Use the angle and the horizontal distance to calculate the unknown height of the object.
- Calculate Unknown Angle (Extension):** Use the calculated height and the measured distance to determine the length of the Hypotenuse (the line of sight), and then use that data to calculate the secondary unknown angle (the angle at the top of the object).

**Success Criteria for Project:** The final report (written or verbal) must include the original sketch, the chosen ratio, and the final calculated height.

## Phase 3: Conclusion and Assessment

### Recap and Reflection (10 minutes)

Review the applications learned today: calculating heights, slopes, and distances. Emphasize that trigonometry gives us power over measurements that are otherwise physically impossible.

**Discussion Prompt:** "Besides surveying and construction, where else might pilots, navigators, or even video game programmers use these ratios?" (Answers could include: determining flight paths, satellite orbit, calculating 3D camera angles).

### Summative Assessment (5 Minutes)

### Exit Ticket:

Complete the following two prompts:

1. If you know the Hypotenuse and the Angle, and you need to find the Opposite side, you must use the \_\_\_\_\_ ratio. (Sine / Cosine / Tangent)
2. Describe one unique real-world problem you solved today (or could solve) using trig ratios.

## Differentiation and Adaptability

### Scaffolding (For learners needing support)

- Provide a printed flow chart specifically detailing when to use SIN, COS, or TAN based on what information is Known and Unknown.
- For the "I Do" and "We Do" sections, use only whole-number angles (30, 45, 60) to keep the initial calculator work simpler and allow focus on the setup.
- Allow the use of a simple online trig calculator that automates the inverse functions ( $\sin^{-1}$ ,  $\cos^{-1}$ ,  $\tan^{-1}$ ).

### Extension (For advanced learners)

- **Challenge Problem:** Introduce the concept of "Angle of Depression." Create a word problem involving a person looking down from a cliff. (This simply flips the reference angle and reinforces the concept of parallel lines and alternate interior angles).
- **3D Application:** Challenge the learner to use trigonometry to calculate the length of the diagonal across a rectangular room from the floor in one corner to the ceiling in the opposite corner (requires the Pythagorean theorem first, then trigonometry).
- **Inverse Modeling:** Task the student with creating a detailed, step-by-step instructional guide (or video) explaining how to use a smartphone inclinometer app to measure height using trigonometry.