

Mission: Impossible? Measuring the Unmeasurable with Trigonometry

Materials Needed

- Scientific calculator (or a calculator app with sin, cos, tan functions)
- Measuring tape or a long ruler
- A smartphone with a "Clinometer" app OR a homemade clinometer (protractor, string, and a washer/weight)
- Writing materials (paper and pencil)
- "The Field Mission" worksheet (included in the 'You Do' section)

Learning Objectives

By the end of this lesson, you will be able to:

- Identify which trigonometric ratio (Sine, Cosine, or Tangent) to use based on given information.
- Calculate the height of a tall object (like a tree or building) using the Tangent ratio.
- Solve real-world distance problems involving right-angled triangles with 95% accuracy.

1. Introduction: The "Spy" Hook (5-10 Minutes)

The Scenario: Imagine you are a secret agent. You need to drop a cable from the top of a high-security building to extract a teammate. You can't exactly walk up to the guards with a 100-foot ladder to measure the height of the wall. You are stuck on the ground 50 feet away. How do you find out exactly how much cable you need without getting caught?

The Secret Weapon: Trigonometry. It's essentially a "cheat code" for the universe. If you know just one angle and one side of a right triangle, you can figure out every other dimension of that triangle. Today, we're going to master the **SOH CAH TOA** method to solve real-world problems.

2. Content & Practice: The "I Do, We Do, You Do" Model

Step 1: The "I Do" - Reviewing the Toolkit (Teacher Modeling)

To solve any right-triangle mystery, we use three ratios. Think of them as the three tools in your belt:

- **SOH:** $\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$
- **CAH:** $\cos(\theta) = \text{Adjacent} / \text{Hypotenuse}$
- **TOA:** $\tan(\theta) = \text{Opposite} / \text{Adjacent}$

Example Problem: A 15-foot ladder leans against a wall. The bottom of the ladder is 5 feet from the wall. What is the angle the ladder makes with the ground?

1. **Identify the sides:** The ladder is the *Hypotenuse* (15). The distance on the ground is the *Adjacent* side (5).

2. **Pick the tool:** We have A and H. Looking at our toolkit, that's **CAH** (Cosine).
3. **Set it up:** $\cos(\theta) = 5 / 15$.
4. **Solve:** $\theta = \cos^{-1}(0.333)$. Using a calculator, $\theta \approx 70.5^\circ$.

Step 2: The "We Do" - The Skateboard Ramp Challenge (Collaborative Practice)

Let's solve this one together. Suppose you want to build a skateboard ramp. You want the angle of the ramp to be 20° for a smooth ride. You know the ramp needs to be 4 feet high (the opposite side).

Discussion Questions:

- If we want to know how *long* the surface of the ramp (the slope) should be, which side of the triangle are we looking for? (Answer: Hypotenuse)
- Which ratio uses "Opposite" and "Hypotenuse"? (Answer: Sine/SOH)
- **The Setup:** $\sin(20^\circ) = 4 / x$.
- **The Algebra:** Multiply both sides by x , then divide by $\sin(20^\circ)$. $x = 4 / \sin(20^\circ)$.
- **The Calculation:** Use your calculator. (Answer: ~ 11.7 feet).

Step 3: The "You Do" - The Field Mission (Independent Application)

Now it's your turn. You are going to measure something in your actual environment (a tree, your house, a basketball hoop) that is too tall to measure by hand.

The Mission Instructions:

1. **Step Outside:** Find a tall object.
2. **Measure Distance:** Walk a certain distance away from it (e.g., 20 feet). This is your **Adjacent** side.
3. **Measure the Angle:** Use your clinometer (or app) to look at the very top of the object. Record the angle of elevation.
4. **Calculate:** Use the **TOA** (Tangent) ratio. $\tan(\text{Angle}) = \text{Height} / \text{Distance}$.
5. **Adjust for Yourself:** Don't forget to add your own height (eye level) to the final answer!

3. Conclusion: Recap & Success Criteria

Recap: Today we moved from "math on paper" to "math in the world." We used SOH CAH TOA to turn angles into distances.

Success Check:

- Can you explain what SOH CAH TOA stands for without looking?
- Do you know why we use Tangent most often for measuring buildings? (Because we usually know the ground distance, but not the diagonal distance).
- Did your "Field Mission" height seem realistic?

4. Assessment

Formative (During the lesson): Participation in the "We Do" skateboard ramp calculation and correct

identification of the trig ratios in the "I Do" section.

Summative (The Result): Submit the "Field Mission" results. The student must show the diagram, the formula used, the calculator steps, and the final height with units (feet/meters).

5. Differentiation & Adaptability

- **For the Tech-Savvy:** Use Google Earth to find a famous landmark (like the Eiffel Tower). Use the "measure" tool to find your distance from the base and find the height using a given angle of elevation found online.
- **For the Struggling Learner:** Focus strictly on the Tangent (TOA) ratio first, as it is the most common for "height" problems, before introducing Sine and Cosine. Use a "Trig Ratio Cheat Sheet" with the algebra steps (e.g., "To find the top number, multiply; to find the bottom number, divide").
- **Extension (Advanced):** Calculate the "Angle of Depression." If you are at the top of a 100ft lighthouse and see a boat, and your angle of sight is 15° downward, how far away is the boat?