The Luck Factor: Mastering Probability and the Monty Hall Paradox

Lesson Overview

In this lesson, we will transition from "guessing" to "predicting." Students will explore the math behind chance, conduct experiments to see how data changes with more attempts, and tackle one of the most famous (and brain-breaking) math puzzles in history: The Monty Hall Problem.

Materials Needed

- 3 Opaque cups (or 3 empty boxes/envelopes)
- A small "prize" (a coin, a candy, or a small toy)
- A deck of cards or a standard 6-sided die
- Paper and pen for tracking data
- A calculator (optional)

Learning Objectives

By the end of this lesson, the learner will be able to:

- Define **Probability** as the likelihood of an event happening.
- Calculate the difference between **Theoretical Probability** (what should happen) and **Experimental Probability** (what actually happens).
- Explain the logic behind the "Monty Hall Problem" using data collected during the lesson.

1. Introduction: The Hook (5 Minutes)

The Scenario: Imagine you are on a game show. The host, Monty, shows you three closed doors. Behind one door is a brand-new mountain bike! Behind the other two? Stinky old socks. You pick Door #1. Monty, who knows what is behind all the doors, opens Door #3 to reveal a stinky sock. He then asks: "*Do you want to stay with Door #1, or switch to Door #2?*"

The Big Question: Does it actually matter if you switch? Or are your chances 50/50 either way? Most people think it doesn't matterâ€”but math says otherwise! Today, weâ€™re going to find out why.

2. Body: Content & Practice

Phase 1: "I Do" - Understanding the Basics (10 Minutes)

The Concept: Probability is usually written as a fraction: *(Number of ways to win) / (Total number of possibilities)*.

- **Theoretical Probability:** If I flip a coin, there are 2 sides. The chance of heads is 1 out of 2, or

50%. This is what *should* happen in a perfect world.
- **Experimental Probability:** This is what happens when we actually do the work. If I flip a coin 10 times, I might get 7 heads and 3 tails. My experimental probability for heads was 70%.
- **The Rule of Large Numbers:** The more times we repeat an experiment, the closer the "Experimental" result will get to the "Theoretical" result.

Phase 2: "We Do" - The Dice Challenge (15 Minutes)

Let's test the math together. We want to see how many times we roll a "4" on a six-sided die.

1. **Predict:** What is the theoretical probability of rolling a 4? (Answer: 1/6 or about 16.6%).
2. **Roll:** Roll the die 12 times. Keep a tally of how many 4s you get.
3. **Calculate:** Divide your number of 4s by 12. Is it exactly 16.6%? Probably not!
4. **Expand:** Now roll it 24 more times. Add those to your previous tally. Is the percentage getting closer to 16.6% now?

Phase 3: "You Do" - Cracking the Monty Hall Problem (20 Minutes)

Now, letâ€™s solve the game show mystery through a simulation. You will act as the "Contestant" and a partner (or your teacher/parent) will act as the "Host."

The Setup: Place the 3 cups upside down. The Host hides the "prize" under one cup while the Contestant looks away.

The Experiment: Run 20 rounds of the game.

- **Rounds 1-10 (The "Stay" Strategy):** You pick a cup. The Host opens one of the *other* empty cups. You **always stay** with your original choice. Record how many times you win.
- **Rounds 11-20 (The "Switch" Strategy):** You pick a cup. The Host opens one of the *other* empty cups. You **always switch** to the remaining closed cup. Record how many times you win.

The Analysis: Look at your data. Which strategy won more often? (Spoiler: Mathematically, switching wins 66% of the time, while staying only wins 33% of the time!)

3. Conclusion: Closure & Recap (10 Minutes)

Summary: Why does switching work? When you first picked, you had a 1/3 chance of being right. That means there was a 2/3 chance the prize was in the *other* two cups. By opening one empty cup, Monty "concentrated" that 2/3 chance into the one remaining cup you didn't pick!

Recap Questions:

- What is the difference between theoretical and experimental probability?
- In the Monty Hall problem, why does the probability change after a door is opened?
- How can we make experimental probability more accurate? (Answer: Do more trials!)

Success Criteria

You have mastered this lesson if you can:

- Explain to someone else why "switching" doors gives you a better chance of winning.
- Calculate the probability of a simple event (like drawing an Ace from a deck of cards).
- Show your recorded data table comparing the "Stay" vs. "Switch" strategies.

Differentiation & Extensions

- **For a Challenge:** Create a "Tree Diagram" to show all possible outcomes of the Monty Hall problem to prove the 2/3 win rate visually.
- **For Extra Support:** Use a deck of only 4 cards (three 2s and one Ace). Practice finding the Ace to make the fractions easier to visualize (1/4 chance vs 3/4 chance).
- **Digital Option:** Use an online "Monty Hall Simulator" to run the experiment 1,000 times in one second to see the Rule of Large Numbers in action.

Assessment

Formative: Observation of the student during the Dice Challenge and their ability to calculate the initial fractions.

Summative: The "Strategy Data Sheet." The student must present their win/loss tally for the Monty Hall simulation and write a one-sentence conclusion based on their specific results.