

# The Sky's the Limit: Mastering Right Triangle Trigonometry

**Subject:** 10th Grade Geometry / Algebra 2

**Duration:** 60-75 Minutes

**Context:** Homeschool, Classroom, or Independent Study

## Lesson Overview

This lesson transforms abstract trigonometric ratios into a practical tool for measuring the world. Students will move from understanding the basic SOH CAH TOA ratios to physically measuring "inaccessible heights" (like trees, buildings, or flagpoles) using a DIY clinometer and right-triangle math.

## Learning Objectives

By the end of this lesson, the learner will be able to:

- Identify the **Opposite**, **Adjacent**, and **Hypotenuse** sides of a right triangle relative to a given angle.
- Define and calculate **Sine**, **Cosine**, and **Tangent** ratios.
- Use the **Tangent** ratio to calculate the height of a real-world object using an angle of elevation and a base distance.

## Materials Needed

- Scientific calculator (or a calculator app with *sin*, *cos*, *tan* functions)
- Measuring tape or a long ruler
- **To make a DIY Clinometer:** A protractor, a drinking straw, about 12 inches of string, and a small weight (like a washer, nut, or paperclip)
- Tape
- Pencil and paper (or a tablet for sketching)

## 1. The Hook: The "Unreachable" Challenge (5 Minutes)

**Scenario:** You are a surveyor tasked with measuring the height of a massive pine tree (or the local water tower) for a construction project. You can't climb it with a tape measure, and you don't have a drone. How do you find the exact height without ever leaving the ground?

**The Answer:** You use the relationship between angles and distances. If you know how far you are standing from the base and the angle at which you are looking at the top, the math does the climbing for you.

## 2. Direct Instruction: SOH CAH TOA (15 Minutes)

### The "I Do" Phase:

Trigonometry is just the study of "triangle measurements." For any right triangle, the ratios of the sides are constant for a given angle. We use the acronym **SOH CAH TOA** to remember them:

- **Sine** = **O**pposite / **H**ypotenuse
- **Cosine** = **A**djacent / **H**ypotenuse
- **Tangent** = **O**pposite / **A**djacent

**Key Concept:** When finding heights of objects on the ground, we almost always use **Tangent** because we know the distance on the ground (Adjacent) and want to find the height (Opposite).

**Example:** If you are 20 feet from a wall and looking up at a 30° angle:

$$\tan(30^\circ) = \text{Height} / 20$$

$$\text{Height} = 20 * \tan(30^\circ)$$

## 3. Guided Practice: Building the Tool (15 Minutes)

### The "We Do" Phase:

Before we go outside, we need a way to measure the "Angle of Elevation." We will build a **Clinometer**.

1. **Assembly:** Tape the straw along the straight edge of the protractor.
2. **The Plumb Line:** Poke a small hole through the center hole of the protractor (the vertex). Tie the string through it and let it hang down. Attach the weight to the bottom of the string.
3. **How to Read It:** Look through the straw at the top of an object. The string will hang straight down. If the string crosses the 60° mark, your angle of elevation is  $90^\circ - 60^\circ = 30^\circ$ .
4. **Check:** Practice looking at a ceiling corner. Have a partner/parent check the string reading.

## 4. Application: The Inaccessible Height Lab (25 Minutes)

### The "You Do" Phase:

Step outside (or use a large indoor space like a gym or hallway). Choose a tall object (a tree, a basketball hoop, or a house peak).

1. **Measure the Base (d):** Use the tape measure to find the distance from where you are standing to the base of the object. Record this as your *Adjacent* side.
2. **Measure the Angle (θ):** Look through your clinometer at the very top of the object. Note the angle. (Remember:  $90 - [\text{Protractor Reading}] = \text{your angle}$ ).
3. **Measure your Eye Level (h1):** Measure the distance from the ground to your eye. (This is important because the triangle starts at your eye, not the ground!)
4. **Calculate:**
  - Use the formula:  $\text{Object Height} = (d * \tan(\theta)) + h1$
5. **Verify:** If possible, measure a smaller object (like a 10ft pole) manually to see how close your trig calculation was.

## 5. Closure and Recap (10 Minutes)

**Summary:** Today we moved from "SOH CAH TOA" on a worksheet to using it as a real-world tool. You learned that if you have a single angle and a single side, you can unlock every other dimension of a right triangle.

### Recap Questions:

- Why did we add our eye-level height at the end? (*Because the triangle we measured sat on top of our own height.*)
- If you move further away from the object, does the height change? Does the angle change? (*Height stays same, angle decreases.*)
- Which ratio would you use if you knew the length of a ladder leaning against a wall (Hypotenuse) and wanted to find how high it reaches? (*Sine.*)

## Assessment

**Formative Assessment:** Observation of the clinometer build and the "check" measurement of the ceiling corner.

**Summative Assessment:** The "Field Note" calculation. The student must turn in a sketch of their triangle, labeled with their measured distance, angle, and the final calculated height of their chosen object. Success is defined as a logically sound calculation, even if physical measurement error is present.

## Differentiation

- **For Struggling Learners:** Provide a pre-printed "Trig Cheat Sheet" showing the triangle labeled with O, A, and H. Use a  $45^\circ$  angle for the experiment (since  $\tan(45^\circ) = 1$ , the height will simply be the distance + eye level).
- **For Advanced Learners:** Challenge them to find the height of an object where they *cannot* reach the base (e.g., a tree on the other side of a fence) using two different angle measurements from two different distances (Law of Sines or systems of equations).

## Success Criteria

- Student can correctly identify the "Adjacent" side as the ground distance.
- Student can correctly convert the protractor reading to the angle of elevation.
- Student uses the "Tangent" function correctly on a calculator.
- The final height estimate is realistic for the object chosen.