Mission: Curved Dimensions – Cylinders and Spheres

Materials Needed

- Scientific calculator (with a π button)
- Measuring tape or ruler (metric preferred)
- At least one cylindrical object (e.g., a soda can, soup tin, or oatmeal container)
- At least one spherical object (e.g., a tennis ball, orange, or basketball)
- String (for measuring circumference)
- Paper and pencil

Learning Objectives

- Identify and measure the radius, diameter, and height of curved solids.
- Calculate the Volume and Surface Area of cylinders.
- Calculate the Volume and Surface Area of spheres.
- Understand the real-world application of these formulas in packaging and design.

Success Criteria

- I can explain the difference between radius and diameter.
- I can correctly select and apply the formulas for volume and surface area.
- I can provide answers with correct units (e.g., cm^2 for area, cm^3 for volume).
- I can round my answers to two decimal places consistently.

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1. Introduction: The Space Cargo Challenge (8 Minutes)

The Hook: Imagine you are an engineer for a private space company. Every square centimeter of material costs $1,000 to produce, and every cubic centimeter of fuel needed to lift weight costs $500. You are tasked with designing storage for "Oxygen Spheres" and "Fuel Cylinders." If your math is off, the mission fails, and the budget explodes!

Discussion Questions:

- Why do we use cylinders for liquids (like soda or fuel) instead of boxes?
- If you have a ball and a cube of the same width, which one do you think holds more "stuff" inside?

The Goal: Today, we master the math of the curve so you can build the most efficient storage possible.

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2. "I Do": Breaking Down the Formulas (12 Minutes)

To master these shapes, we need to understand where the formulas come from. We treat π (pi) as roughly 3.14159, but we always use the calculator button for precision.

A. The Cylinder

Think of a cylinder as a stack of circles.

- **Volume (V):** Area of the base $\times$ height. $V = \pi r^2 h$
- **Surface Area (SA):** Two circles (top and bottom) + the "label" part (which is just a rectangle wrapped around). $SA = 2\pi r^2 + 2\pi rh$

B. The Sphere

Spheres are unique because they only have one dimension: the radius (r).

- **Volume (V):** $V = \frac{4}{3} \pi r^3$
- **Surface Area (SA):** $SA = 4 \pi r^2$ (Interestingly, this is exactly the area of four circles with the same radius!)

Pro-Tip: Always identify your radius first. If you have the diameter (the whole way across), divide it by 2!

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3. "We Do": Guided Practice (10 Minutes)

Let’s solve one together. Grab your calculator.

The Scenario: We have a standard "Star-Fuel" canister. It is a cylinder with a **radius of 5cm** and a **height of 12cm**.

1. **Volume:** $V = \pi \times 5^2 \times 12$.
   - Calculation: $\pi \times 25 \times 12 = 300\pi$.
   - Final Answer: $\approx 942.48 cm^3$.
2. **Surface Area:** $SA = (2 \times \pi \times 5^2) + (2 \times \pi \times 5 \times 12)$.
   - Calculation: $50\pi + 120\pi = 170\pi$.
   - Final Answer: $\approx 534.07 cm^2$.

Check for Understanding: What happens to the volume if we double the height? What if we double the radius? (Hint: One is much more powerful!)

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4. "You Do": The Real-World Lab (20 Minutes)

Now it's your turn to be the lead engineer. Complete the following tasks using the objects you gathered.

Task 1: The Cylinder Audit

1. Measure the height of your cylinder.
2. Measure the diameter and divide by 2 to find the radius.
3. Calculate the total Volume.
4. Calculate the Surface Area (the total amount of material used to make the can).

Task 2: The Sphere Audit

1. Find the radius of your sphere. (Tip: If it’s hard to measure the middle, wrap a string around the widest part to find the circumference, then use $r = \frac{C}{2\pi}$).
2. Calculate the Volume.
3. Calculate the Surface Area.

Task 3: The Design Challenge

You need to design a spherical capsule that holds exactly $2,000 cm^3$ of medical supplies. Use your algebra skills to find the required radius.
Scaffold: $2000 = \frac{4}{3} \pi r^3$. Solve for r.

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5. Conclusion & Recap (8 Minutes)

Summary: Today we moved from flat shapes into the 3D world of curves. We learned that cylinders are just "stacked circles" and spheres are incredibly efficient containers.

The "Quick-Fire" Review:

- Which formula uses r^3? (Volume of a sphere)
- Which formula includes $2\pi rh$? (Side surface area of a cylinder)
- If a sphere and a cylinder have the same radius and the cylinder's height is equal to its diameter, which one is bigger? (The cylinder!)

Final Reflection: In your notebook, write one way that knowing the surface area of a sphere could help a company like NASA or a manufacturer like Nike.

Differentiation Options

- **For Struggling Learners:** Provide a "Cheat Sheet" with the steps: 1. Find r, 2. Square or Cube r, 3. Multiply by π, 4. Multiply by height or constant. Use a calculator for all π operations.
- **For Advanced Learners:** Challenge them with a "Composite Solid." Calculate the volume of a "Silo": a cylinder with a hemisphere (half-sphere) on top. How does the math change?

Assessment Methods

- **Formative:** Observation during the "We Do" section and checking the measurements/calculations during the "You Do" lab.
- **Summative:** The Design Challenge (Task 3) serves as a check for algebraic manipulation and understanding of the volume formula.