Instructions

Answer all questions in the spaces provided. Show all your working. You may use a calculator for Section 2. Relevant formulas are provided where necessary.

Section 1: Algebra, Indices & Equations

1. Simplify the following expressions, leaving your answers with positive indices.

- a) $(3x^{-4}y^2)^3$
- b) $(64m^6n^9)^{2/3}$
- c) $\frac{(2a^5b)^2}{4a^{12}b^{-1}}$

2. Simplify the following expressions involving surds.

- a) $\sqrt{72} + \sqrt{50}$
- b) $(2\sqrt{3} - 1)(\sqrt{3} + 5)$

3. Expand and factorise the following algebraic expressions.

- a) Expand: $(4x - 3)(2x + 5)$
- b) Factorise: $x^2 + 2x - 24$
- c) Factorise: $9x^2 - 49$

4. Solve the following linear equation.

$$\frac{4(x - 3)}{5} = 2x - 13$$

5. Solve the following non-monic quadratic equation using any method. Show your working.

$$3x^2 - 11x + 6 = 0$$

6. Solve the following simultaneous equations.

$$4x + 3y = 18$$
$$x - 2y = -1$$

Section 2: Surface Area & Volume

Formulas you may need:

- **Cone:** Volume = $\frac{1}{3}\pi r^2 h$, Surface Area = $\pi rs + \pi r^2$ *(where s is the slant height)*
- **Sphere:** Volume = $\frac{4}{3}\pi r^3$, Surface Area = $4\pi r^2$
- **Pythagoras' Theorem:** $a^2 + b^2 = c^2$

For this section, you can leave answers in terms of π or round to two decimal places.

7. A right-angled cone has a radius of 8 cm and a perpendicular height of 15 cm.

- a) First, find the slant height, *s*, of the cone.
- b) Calculate the volume of the cone.
- c) Calculate the total surface area of the cone.

8. A solid object is formed by a hemisphere perfectly placed on top of a cylinder. The radius of both the cylinder base and the hemisphere is 5 m. The height of the cylinder is 12 m.

> **Solid object description:**
>
> - Shape: Hemisphere on a cylinder
> - Radius (both parts): 5 m
> - Cylinder height: 12 m

- a) Calculate the total volume of the solid object.
- b) Calculate the total external surface area of the object. (This includes the circular base of the cylinder, the curved side of the cylinder, and the curved surface of the hemisphere).

Section 3: Coordinate Geometry

Formulas you may need:

- **Distance:** $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
- **Midpoint:** $M = \left(\frac{(x_1 + x_2)}{2} , \frac{(y_1 + y_2)}{2} \right)$
- **Gradient:** $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

9. Consider the two points A(-4, 9) and B(2, 1).

- a) Calculate the exact distance between points A and B. Leave your answer as a simplified surd.
- b) Find the gradient of the line segment AB.

- c) Find the coordinates of the midpoint of the line segment AB.

Answer Key

Section 1: Algebra, Indices & Equations

1.
a) $(3x^{-4}y^2)^3 = 3^3x^{-12}y^6 =$ **$27y^6 / x^{12}$**
b) $(64m^6n^9)^{2/3} = (\sqrt[3]{64})^2 * m^{(6 * 2/3)} * n^{(9 * 2/3)} = 4^2m^4n^6 =$ **$16m^4n^6$**
c) $\dfrac{(2a^5b)^2}{4a^{12}b^{-1}} = \dfrac{4a^{10}b^2}{4a^{12}b^{-1}} = a^{(10-12)}b^{(2 - (-1))} = a^{-2}b^3 =$ **b^3 / a^2**

2.
a) $\sqrt{72} + \sqrt{50} = \sqrt{(36 \times 2)} + \sqrt{(25 \times 2)} = 6\sqrt{2} + 5\sqrt{2} =$ **$11\sqrt{2}$**
b) $(2\sqrt{3} - 1)(\sqrt{3} + 5) = 2\sqrt{3} \times \sqrt{3} + 2\sqrt{3} \times 5 - 1 \times \sqrt{3} - 1 \times 5 = 2(3) + 10\sqrt{3} - \sqrt{3} - 5 = 6 - 5 + 9\sqrt{3} =$ **$1 + 9\sqrt{3}$**

3.
a) $(4x - 3)(2x + 5) = 8x^2 + 20x - 6x - 15 =$ **$8x^2 + 14x - 15$**
b) $x^2 + 2x - 24 =$ **$(x + 6)(x - 4)$**
c) $9x^2 - 49 = (3x)^2 - 7^2 =$ **$(3x - 7)(3x + 7)$**

4.
$4(x - 3) = 5(2x - 13)$
$4x - 12 = 10x - 65$
$-12 + 65 = 10x - 4x$
$53 = 6x$
$x = 53/6$ or **$8 \frac{5}{6}$**

5.
$3x^2 - 11x + 6 = 0$
$(3x - 2)(x - 3) = 0$
$3x - 2 = 0$ or $x - 3 = 0$
$x = 2/3$ or **$x = 3$**

6.
(1) $4x + 3y = 18$
(2) $x - 2y = -1 \Rightarrow x = 2y - 1$
Substitute (2) into (1):
$4(2y - 1) + 3y = 18$
$8y - 4 + 3y = 18$
$11y = 22$
$y = 2$
Substitute $y = 2$ into $x = 2y - 1$:
$x = 2(2) - 1 = 4 - 1$
$x = 3$
Solution: **$x = 3, y = 2$**

Section 2: Surface Area & Volume

7.
a) $s^2 = r^2 + h^2 = 8^2 + 15^2 = 64 + 225 = 289$. $s = \sqrt{289} =$ **17 cm**.
b) Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 8^2 \times 15 = \frac{1}{3} \times \pi \times 64 \times 15 = 320\pi\ cm^3$ ≈ **$1005.31\ cm^3$**.
c) Surface Area $= \pi rs + \pi r^2 = \pi(8)(17) + \pi(8)^2 = 136\pi + 64\pi =$ **$200\pi\ cm^2$** ≈ **$628.32\ cm^2$**.

8.
a) Volume $= Vol_{cylinder} + Vol_{hemisphere}$
$Vol_{cylinder} = \pi r^2 h = \pi(5)^2(12) = 300\pi\ m^3$.
$Vol_{hemisphere} = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi(5)^3 = \frac{2}{3} \times 125\pi = \frac{250}{3}\pi\ m^3$.
Total Volume $= 300\pi + \frac{250}{3}\pi = \frac{900\pi+250\pi}{3} =$ **$\frac{1150}{3}\pi\ m^3$** ≈ **$1204.28\ m^3$**.

b) Surface Area $= SA_{cylinder\ base} + SA_{cylinder\ side} + SA_{hemisphere}$
$SA_{cylinder\ base} = \pi r^2 = \pi(5)^2 = 25\pi\ m^2$.
$SA_{cylinder\ side} = 2\pi rh = 2\pi(5)(12) = 120\pi\ m^2$.
$SA_{hemisphere} = \frac{1}{2} \times 4\pi r^2 = 2\pi(5)^2 = 50\pi\ m^2$.
Total Surface Area $= 25\pi + 120\pi + 50\pi =$ **$195\pi\ m^2$** ≈ **$612.61\ m^2$**.

Section 3: Coordinate Geometry

9. For A(-4, 9) and B(2, 1):
a) Distance $= \sqrt{[(2 - (-4))^2 + (1 - 9)^2]} = \sqrt{[(6)^2 + (-8)^2]} = \sqrt{[36 + 64]} = \sqrt{100} =$ **10 units**.
b) Gradient $= \frac{(1 - 9)}{(2 - (-4))} = \frac{-8}{6} =$ **$-\frac{4}{3}$.**
c) Midpoint $= \left(\frac{(-4 + 2)}{2}, \frac{(9 + 1)}{2} \right) = \left(\frac{-2}{2}, \frac{10}{2} \right) =$ **(-1, 5)**.